MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2017/2018

BMS1024 – MANAGERIAL STATISTICS

(All sections / Groups)

1 June 2018
3.00 pm – 5.00 pm
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **THIRTEEN (13)** printed pages with:
   Section A: Ten (10) multiple choice questions (20%)
   Section B: Three (3) structured questions (80%)

2. Answer **ALL** questions.

3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.

4. Statistical tables are attached at the end of the question paper.

5. Students are allowed to use non-programmable scientific calculators with no restriction.

SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

There are TEN (10) questions in this section. Answer ALL questions on the multiple choice answer sheet.

1. One-fourth of the data falls below the _____ quartile.
   A. fourth
   B. second
   C. first
   D. third

2. What is the probability of choosing a day of the week that begins with T (Tuesday or Thursday)?
   A. 2/7
   B. 1/49
   C. 7/2
   D. 5/7

3. In testing the hypothesis: H_0: $\mu \geq 150$ and H_1: $\mu < 150$, the value of Z-test statistic is -2.42. The *p-value* is:
   A. -2.42
   B. 2.42
   C. 0.0078
   D. 0.9922

4. Suppose that a one-tail t test is being applied to find out if the populations mean less than 100. The level of significance is 5% and 25 observations were sampled. The rejection region is:
   A. $t < -1.711$
   B. $t > 1.708$
   C. $t = 2.064$
   D. $t < -1.316$

5. Hypothesis testing is a technique that allows you to draw statistically valid conclusions about _____ .
   A. random errors
   B. population parameters
   C. sample statistics
   D. individuals

Continued...

6. Given that the standard deviation is equal to 0.568, the median equals 5 and the mean value is 3.5, what is the value of the coefficient of variation?
   A. 350%
   B. 500%
   C. 56.8%
   D. 16.23%

7. The _____ is the observation that occurs the most frequently in the data set.
   A. mode
   B. median
   C. standard deviation
   D. range

8. Which of the following is not a characteristic for a normal distribution?
   A. It is a bell-shaped distribution.
   B. The probability is measured by the area above the curve.
   C. It is symmetrical.
   D. The mean is roughly equal to the median.

9. Identify the correct statement from the following :
   A. If two events, A and B, are mutually exclusive, then P(A or B) = P(AB)
   B. The probability of the complement of any event A is 1 – P(A)
   C. If two events, A and B, are mutually exclusive, then P(A or B) = P(A) – P(B)
   D. If two events, A and B, are not mutually exclusive, then P(A or B) = P(A) + P(B)

10. The degrees of freedom for the test statistic for μ when σ is unknown is:
    A. 1
    B. n
    C. n - 2
    D. n - 1

Continued...

SECTION B: STRUCTURED QUESTIONS (80 MARKS)

There are THREE questions in this section. Candidates MUST answer ALL THREE questions.

QUESTION 1 (25 Marks)

a) A hotel resort in Pulau Tioman has the following type of accommodation:

Number of Bedrooms

Location	3 Bedrooms	4 Bedrooms	Total
Beach Front	17	24	41
Non Beach Front	6	210	216
Total	23	234	257

i) What is the probability of selecting a beach front site?

ii) What is the probability of selecting a non beach front site with 3 bedrooms?

iii) What is the probability of selecting a non beach front site given that the facility had 4 bedrooms?

iv) Is the event of selecting a beach front site independent of the event of selecting a 3 bedroom facility? Explain .

[4 + 2 + 4 + 5 = 15 marks]

b) A road traffic survey in West Malaysia revealed that 60% of the vehicles traveling on the North-South Expressway, were exceeding the speed limit, where the speed limit is 110 km per hour. In a random selection of ten vehicles, find the following probabilities, where x denote the number of vehicles that were exceeding the speed limit.

i) $P(x = 10)$
ii) $P(4 < x < 7)$
iii) $P(3 \leq x \leq 4)$

[2 + 4 + 4 = 10 marks]

Continued...

QUESTION 2 (25 Marks)

a) A manufacturer of salad dressings uses machines to dispense liquid ingredients into bottles that move along a filling line. The machine that dispenses dressings is working properly when 8 ounces are dispensed. The standard deviation of the process is 0.15 ounces. If the average amount dispensed in a particular sample of 50 bottles is 7.952 ounces, find a 99% confidence interval around this sample mean.

[5 marks]

b) The sales analyst wants to estimate the average number of bottled drinks sold each day out of the vending machine. Similar vending machine sales have a standard deviation of 9 bottles per day. She wants to be 95-percent confident that the mean estimate ($\bar{x}$), obtained is within 3 bottles of the true mean μ. How large a sample is needed?

[4 marks]

c) A light bulb manufacturer claims that the average life of their light bulb is more than 1000 hours. To test this claim, a sample of 32 light bulbs had an average life of 1120 hours. Assume the population standard deviation is 325 hours and use $\alpha = 0.025$.

i) State the null and alternative hypothesis.

ii) Compute the test statistic and state the decision rule in terms of the critical value

iii) What is your conclusion?

[2 + 5 + 1 = 8 marks]

Continued...

d) A regional automotive parts chain store firm wants to improve the sales of tune-up supplies. It believes that a television advertisement with a popular local, but offbeat, know-it-all actor might be able to affect their sales. Before the advertisements are run on television, the company randomly samples eight of its weekly sales from past years. Following the advertisement campaign, eight weeks of sales were sampled. Weekly sales are approximately normally distributed and the population standard deviations are equal. Their hypothesis test is: Did the television advertisement campaign have a significant effect on sales at 0.05 level of significance? The results of the two samples are as follows:

Before Ad Campaign	After Ad Campaign
$n_1 = 8$	$n_2 = 8$
$\bar{X}_1 = 184$	$\bar{X}_2 = 189$
$s_1^2 = 548$	$s_2^2 = 456$

i) State the null and alternative hypotheses.

ii) What is the appropriate test and it's test statistic?

iii) What is the conclusion?

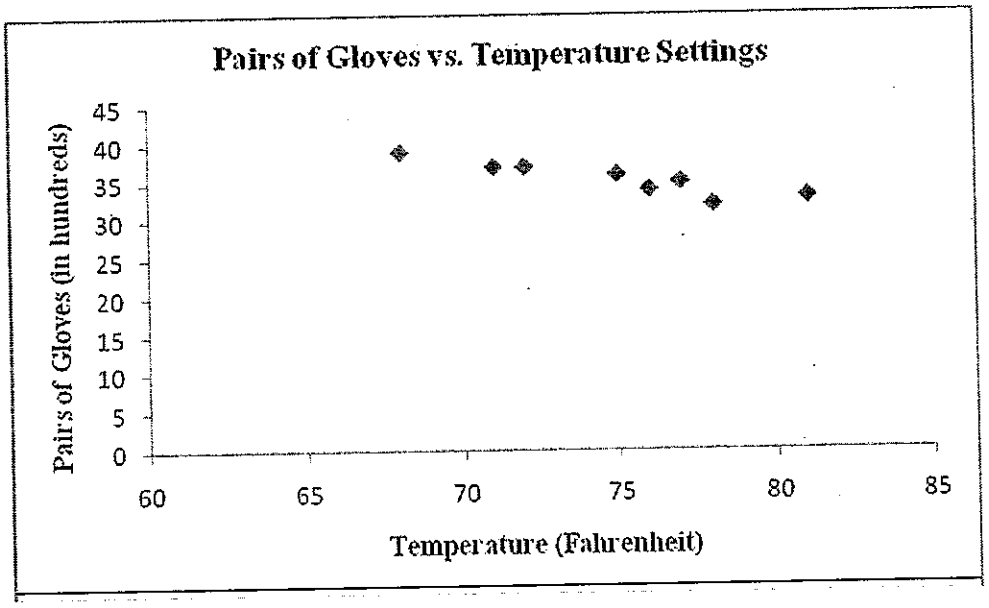
[2 + 5 + 1 = 8 marks]

Continued...

QUESTION 3 (30 Marks)

a) The management of a small factory that produces safety gloves is concerned about the high cost of air conditioning in the summer but is afraid that keeping the temperature in the unit too high will lower productivity. During the summer, the management experiments with temperature settings from 68 to 81 degrees Fahrenheit and measures each day's productivity. The following table gives the temperature and the number of pairs of gloves (in hundreds) produced on each of the eight randomly selected days. The scatter-plot of the relationship between the two variables are also shown below.

Temperature	Pairs of Gloves
72	37
71	37
78	32
75	36
81	33
77	35
68	39
76	34



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Regression Statistics	
Multiple R	-0.925
R Square	0.855
Adjusted R Square	0.831
Standard Error	0.956
Observations	8

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	73.658	6.438	11.441	2.67569E-05
Temperature	–0.512	0.086	–5.954	0.001003844

i) Based on the data plotted on the scatter plot, describe the relationship between the temperature settings and the productivity of the glove factory. **[4 marks]**

ii) Determine the least square regression equation to depict the relationship between the temperature settings and the productivity of the glove factory. **[4 marks]**

iii) Interpret the extent of the influence of the temperature settings on the productivity of the glove factory. **[4 marks]**

iv) Interpret the correlation coefficient between the temperature settings and the productivity of the glove factory. **[4 marks]**

v) Interpret the coefficient of determination between the temperature settings and the productivity of the glove factory. **[4 marks]**

vi) Predict the productivity of the glove factory if the temperature is set at 74° Fahrenheit. Interpret the result. Is this estimate reliable? Why? **[4 marks]**

b) Answer the following questions.

i) Explain two differences between Laspeyres and Paasche Price Indices. **[3 marks]**

ii) Explain the differences between nominal and real income. **[3 marks]**

End of Paper

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Mean $(\bar{x}) = \frac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation $(s) = \sqrt{\frac{\sum_{i=1}^{n} X_i^2}{n-1} - \frac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation $(CV) = \frac{\sigma}{\bar{X}} \times 100$

Pearson's Coefficient of Skewness $(S_k) = \frac{3(\bar{X} - Median)}{s}$

B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B) if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

Poisson Probability Distribution

If X follows a Poisson Distribution , $P(\lambda)$ where $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

Binomial Probability Distribution

If X follows a Binomial Distribution B(n, p) where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

Normal Distribution

If X follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \frac{X - \mu}{\sigma}$

C. EXPECTATION AND VARIANCE OPERATORS

$E(X) = \Sigma[X \bullet P(X)]$

$VAR(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \Sigma[X^2 \bullet P(X)]$

If $E(X) = \mu$ then $E(cX) = c\,\mu$, $E(X_1 + X_2) = E(X_1) + E(X_2)$

If $VAR(X) = \sigma^2$ then $VAR(cX) = c^2\,\sigma^2$,

$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,COV(X_1, X_2)$

where $COV(X_1, X_2) = E(X_1 X_2) - [E(X_1)\,E(X_2)]$

D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

(100 – α) % Confidence Interval for Population Mean (σ Known) = $\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Mean (σ Unknown) = $\mu = \overline{X} \pm t_{\alpha/2,n-1}\left(s/\sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Proportion $= \hat{p} \pm Z_{\alpha/2}\sigma_{\hat{p}}$

Where $\sigma_{\hat{p}} = \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean $= n \geq \left[\dfrac{(Z_{\alpha/2})\sigma}{E}\right]^2$

Sample Size Determination for Population Proportion $= n \geq \dfrac{(Z_{\alpha/2})^2\,\hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

E. HYPOTHESIS TESTING

One Sample Mean Test

Standard Deviation (σ) Known	**Standard Deviation (σ) Not Known**
$Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	$t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

One Sample Proportion Test

$z = \dfrac{\hat{p} - p}{\sigma_p}$ where $\sigma_p = \sqrt{\dfrac{p(1-p)}{n}}$

Two Sample Mean Test

Standard Deviation (σ) Known

$$z = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

Standard Deviation (σ) Not Known

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$$

where $S_p^2 = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{(n_1 + n_2 - 2)}$

Two Sample Proportion Test

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where } p = \frac{X_1 + X_2}{n_1 + n_2}$$

where X_1 and X_2 are the number of successes from each population

F. REGRESSION ANALYSIS

Simple Linear Regression

Population Model: $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

Sample Model: $\hat{y} = b_0 + b_1 x_1 + e$

Correlation Coefficient

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left(\left(\sum X\right)^2 / n\right)\right]\left[\sum Y^2 - \left(\left(\sum Y\right)^2 / n\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

ANOVA Table for Regression

Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	*1*	SSR	MSR = SSR/1
Error/Residual	$n-2$	SSE	MSE = SSE/$(n-2)$
Total	$n-1$	SST	

Test Statistic for Significance of the Predictor Variable

$t_i = \frac{b_i}{S_{b_i}}$ and the critical value = $\pm t_{\alpha/2,(n-p-1)}$

Where p= number of predictor

G. INDEX NUMBERS

Simple Price Index

$P = \frac{p_t}{p_0} \times 100$

Aggregate Price Index

$P = \frac{\sum p_t}{\sum p_0}(100)$

Laspeyres Price Index

$P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$

Paasche Price Index

$P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$

Laspeyres Quantity Index

$P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$

Paasche Quantity Index

$P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$

Fisher's Ideal Price Index

$\sqrt{(Laspeyres\ \ Price\ Index)(Paa\ sche\ Price\ Index)}$

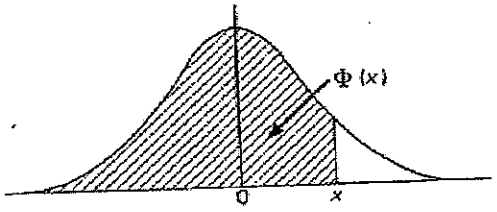
Value Index

$V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$

STATISTICAL TABLE

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\, dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

3·075	0·9990	3·263	0·9994	3·731	0·99990	3·916	0·99995
3·105	0·9991	3·320	0·9995	3·759	0·99991	3·976	0·99996
3·138	0·9992	3·389	0·9996	3·791	0·99992	4·055	0·99997
3·174	0·9993	3·480	0·9997	3·826	0·99993	4·173	0·99998
3·215	0·9994	3·615	0·9998	3·867	0·99994	4·417	0·99999
			0·9999		0·99995		1·00000

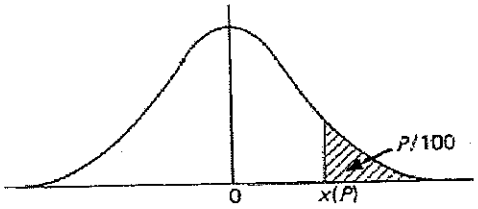
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteq \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geq x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geq x(P)$ is $2P/100$.



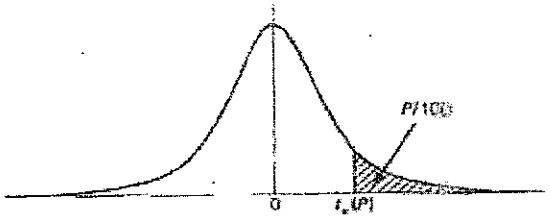
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}} \frac{\Gamma(\frac{1}{2}\nu + \frac{1}{2})}{\Gamma(\frac{1}{2}\nu)} \int_{t_\nu(P)}^{\infty} \frac{dt}{(1 + t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291